Choice based Credit System (CBCS)
Scheme and course structure for
M.A/M.Sc Mathematics $3^{\text {rd }}$ semester effective from academic session 2015 and onwards

| Course Code | Course Name | Credits |
| :--- | :--- | :--- |
| MM14301CR | Ordinary Differential Equations | 4 |
| MM14302CR | Complex Analysis-II | 4 |
| MM14303CR | Functional Analysis-I | 4 |

OPTIONAL COURSES (SEMESTER -III)

| Course Code | Course Name | Credits |
| :--- | :--- | :--- |
| MM14304EA | Abstract Measure Theory | 4 |
| MM14305EA | Advanced Graph Theory | 4 |
| MM14306EA | Mathematical Biology | 4 |
| MM14307EA | Wavelet Theory | 4 |
| MM14308EO | Other Allied + Open | 4 |

## General Instructions for the Candidates

1. The two year ( 4 semester) PG programmes is of 96 credit weightage i.e., 24 credits/semester ( $24 \mathrm{x} 4=96$ ).
2. A candidate has compulsory to opt for 12 credits from the core component in each semester.
3. A candidate has a choice to opt for any 12 credits(3 papers) out of minimum of 16 credits(4 papers) offered as Electives(Allied), except for a particular semester as mentioned by the Department where a candidate is required to gain a minimum of 4 credits( 1 paper) from Elective(Open) offered by any other Department.
4. A candidate has compulsorily to obtain a minimum of 4 credits (1 paper) from Elective (Open) from outside the parent Department in any of the semesters.
5. A candidate can earn more than the minimum required credits (i.e., more than 96 credits for four semester programme) which shall be counted towards the final result of the candidate.

## CREDIT-I

First order ODE, Singular solutions, p-discriminate and c-discriminate, Initial value problem of first order ODE, General theory of Homogeneous and Nonhomogeneous linear ODE, simultaneous linear equations with constant coefficients. Normal form. Factorization of operators. Method of variation of parameters, Picard's theorem on the existence and uniqueness of solutions to an initial value problem.

## CREDIT-II

Solution in Series: (i) Roots of an Indicial equation, un-equal and differing by a quantity not an integer. (ii) Roots of an Indicial equation, which are equal. (iii) Roots of an Indicial equation differing by an integer making a coefficient infinite. (iv) Roots of an Indicial equation differing by an integer making a coefficient indeterminate.
Simultaneous equation $d x / P=d y / Q=d z / R$ and its solutions by use of multipliers and a second integral found by the help of first. Total differential equations $\operatorname{Pdx}+\mathrm{Qdy}+\mathrm{Rdz}=0$. Necessary and sufficient condition that an equation may be integrable. Geometric interpretation of the $\mathrm{Pdx}+\mathrm{Qdy}+\mathrm{Rdz}$ $=0$.

## CREDIT-III

Existence of Solutions, Initial value problem, Ascoli- lemma, Cauchy Piano existence theorem, Uniqueness of solutions with examples, Lipchitz condition and Gronwall inequality, Method of successive approximation, Picard-Lindlof theorem, Continuation of solutions, System of Differential equations, Dependence of solutions on initial conditions and parameters.

CREDIT-IV
Maximal and Minimal solutions of the system of Ordinary Differential equations, Cartheodary theorem, Linear differential equations, Linear Homogeneous equations, Linear system with constant coefficients, Linear systems with periodic coefficients, Fundamental matrix and its properties, Non-homogeneous linear systems, Variation of constant formula. Wronskian and its properties.

## Recommended Books:

1. H.T.H. Piaggio, Differential Equations, CBS Publishers and Distributors, New Delhi.
2. P.Hartmen : Ordinary Differential Equations.
3. W.T.Reid : Ordinary Differential Equations.
4. E.A.Coddington and N.Levinson :Theory of Ordinary Differential Equations.
5. D. Somasundaram, Ordinary Differential Equations, Narosa Publishers, New Delhi.

## CREDIT-I

Calculus of Residues, Cauchy's residue theorem, Evaluation of integrals by the method of residues, Parseval's Identity, Branches of many-valued functions with special reference to $\arg (z)$, $\log z$ and $z^{n}$, Blashke's theorem i.e., if $f(z)$ is analytic and bounded by 1 in $|z| \leq 1$ and vanishes at $z=z_{n}, \mathrm{n}=1,2, \ldots \ldots .,\left|z_{n}\right| \leq 1$, then $\sum\left(1-\left|z_{n}\right|\right)$ is convergent or else $f(z) \equiv 0$.

## CREDIT-II

Poisson integral formula for circle and half plane, Poisson-Jensen formula, Estermann's uniqueness theorem, Carlemann's theorem and the uniqueness theorem associated with it, Hadamard's three circle theorem, Log M(r) and Log $\mathrm{I}_{2}(\mathrm{r})$ as convex functions of $\log \mathrm{r}$, Theorem of Borel and Caratheodory.

## CREDIT-III

Power series: Cauchy-Hadamard formula for the radius of convergence, Picad's theorem on power series: If $\mathrm{a}_{\mathrm{n}}>\mathrm{a}_{\mathrm{n}+1}$ and $\lim \mathrm{a}_{\mathrm{n}}=0$, then the series $\sum a_{n} z^{n}$ has radius of convergence equal to 1 and the series converges for $|z|=1$ except possibly at $z=1$, A power series represents an analytic function within the circle of convergence, Hadamard- Pringsheim theorem. The principle of analytic continuation, uniqueness of analytic continuation, Power series method of analytic continuation, functions with natural boundaries e.g., $\sum z^{n!}, \sum z^{2^{n}}$, Schwarz reflection principle.

## CREDIT-IV

Functions with positive real part, Borel's theorem: If $\mathrm{f}(\mathrm{z})=1+\sum a_{n} z^{n}$ is analytic and has a positive real part in $|z|<1$, then $\left|a_{n}\right| \leq 2$, Univalent functions, Area theorem, Bieberbach's conjecture ( statement only) and Koebe's $1 / 4$ theorem.

Space of Analytic Functions, Bloch's Theorem, Schottky's theorem, a - points of an analytic function, Picard's theorem viz, an integral Function which is not a constant takes every value with one possible exception, Landau's theorem.

## Recommended Books:

1. L.V. Ahlfors, Complex Analysis
2. E. C.Tichtmarsh, Theory of Functions
3. J. B. Conway, Functions of a Complex Variable-I
4. Richard Silverman, Complex Analysis
5. Z. Nehari, Conformal Mappings
6. A.I. Markushevish, Theory of Functions of a Complex Variable

Course No: MM14303CR
Course Name:- Functional Analysis-I
Duration of Examination: 2:30 Hrs.
No. of Credits: 04
BANACH SPACE:

## CREDIT-I

Banach Spaces: Definition and examples, subspaces, quotient spaces, Continuous Linear Operators and their Characterization, Completeness of the space $L(X, Y)$ of bounded linear operators (and its converse), incompleteness of $\mathrm{C}[\mathrm{a}, \mathrm{b}$ ], under the integral norm, Finite dimensional Banach spaces, Equivalence of norms on finite dimensional space and its consequences, Dual of a normed linear space, Hahn Banach theorem (extension form) and its applications, Complemented subspaces, Duals of $l_{p}{ }^{n}, C_{o}, l_{p}(p \geq 1), C[a, b]$.

## CREDIT-II

Uniform boundedness Principle and weak boundedness, Dimension of an $\infty$ dimensional Banach space, Conjugate of a continuous linear operator and its properties, Banach-Steinhauss theorem, open Mapping and closed graph theorems, counterexamples to Banach-Steinhauss, open mapping theorem and closed graph theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces ( $\mathrm{C}_{0}, \mathrm{C}[0,1$ ], $l_{p}, p \geq 1$ ), Reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, Examples of reflexive and non-reflexive Banach spaces.

## HILBERT SPACE:

## CREDIT-III

Hilbert spaces: Definition and examples, Cauchy's Schwartz inequality, Parallogram law, orthonormal (o.n) systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer theorem, Gram Schmidt process, o.n basis in separable Hilbert spaces.

## CREDIT-IV

Projection theorem, Riesz Representation theorem. Counterexample to the Projection theorem and Riesz Representation theorem for incomplete spaces. Hilbert property of the dual of a Hilbert space and counter examples of incomplete inner product spaces, Reflexivity of Hilbert space, Adjoint of a Hilbert space operator, weak convergence and Bolzano-Weirstrass property in Hilbert Spaces. Normal and Unitary operators, Finite dimensional spectral theorem for normal operators.

## Recommended Books:

1. B.V.Limaya: Functional Analysis.
2. C.Goffman G. Pedrick: A First Course in Functional Analysis.
3. L.A. Lusternick \& V.J. Sobolov. : Elements of Functional Analysis.
4. J.B. Conway : A Course in Functional Analysis.

Course No. MM14304EA
Course Name:- Abstract Measure Theory
Duration of Examination: 2:30 Hrs.
No. of Credits: 04

Max. Marks:
100

External Exam: 80
Internal Assessment: 20

## CREDIT-I

Semiring, algebra and $\sigma$ - algebra of sets, Borel sets, measures on semirings, outer measure associated with a set function and basic properties, measurable sets associated with an outer measure as a $\sigma$ - algebra, construction of the Lebesgue measure on $R^{n}$.

## CREDIT-II

For $\mathrm{f} \in \mathrm{L}_{1}[\mathrm{a}, \mathrm{b}], \mathrm{F} /=\mathrm{f}$ a.e. on [a,b]. If f is absolutely continuous on (a, b) with $\mathrm{f}(\mathrm{x})=0$ a.e, then $\mathrm{f}=$ constant. Characterization of an absolutely continuous function as an indefinite Lebesgue integral. Non-Lebesgue integrability of f where $\mathrm{f}(\mathrm{x})=\mathrm{x}^{2} \sin \left(1 / \mathrm{x}^{2}\right), \mathrm{f}(0)=0$ on $[0,1]$. Fundamental theorem of calculus for the Lebesgue integral. A brief introduction to $L_{p}$ spaces. Holder's and Minkowki's inequalities.

## CREDIT-III

Improper Riemann integral as a Lebesgue integral, calculation of some improper Riemann integrable functions, space of Lebesgue integrable functions as completion of Riemann integrable functions on [a,b], change of variables formula and simple consequences, Riemann Lebesgue lemma.

## CREDIT-IV

Product measures and iterated integrals, example of non-integrable functions whose iterated integrals exist (and are equal), Fubini theorem, expressing a double integral as an iterated integral, Tonelli-Hobson theorem as a converse to Fubini theorem, differentiation under the integral sign.

## Recommended Books:

1.C.D.Aliprantis and O.Burkinshaw, Principles of Real Analysis
2.Goldberg, R. : Methods of Real Analysis
3.T.M.Apostol : Mathematical Analysis

## Suggested Readings:

1.Royden, L: Real Analysis (PHI)
2.Chae, S.B. Lebesgue Integration(Springer Verlag).
3.Rudin, W. Principles of Mathematicals Analysis(McGraw Hill).
4.Barra ,De. G. : Measure theory and Integration ( Narosa)
5.Rana ,I.K. : An Introduction to Measure and Integration, Narosa Publications.

## CREDIT-I

## Colorings

Vertex coloring, chromatic number $\chi(G)$, bounds for $\chi(G)$, Brook's theorem, edge coloring, Vizing's theorem, map coloring, six color theorem, five color theorem, every graph is four colourable iff every cubic bridgeless plane map is 4-colorable, every planar graph is 4-colorable iff $\chi^{\prime}(G)=3$, Heawood mapcoloring theorem, uniquely colorable graphs

## CREDIT-II

## Matchings

Matchings and 1-factors, Berge's theorem, Hall's theorem, 1-factor theorem of Tutte, antifactor sets, f-factor theorem, f-factor theorem impliesl-factor theorem, Erdos- Gallai theorem follows from f-factor theorem, degree factors, $k$ factor theorem, factorization of $K_{n}$.

## CREDIT-III

## Edge graphs and eccentricity sequences

Edge graphs, Whitney's theorem on edge graphs, Beineke's theorem, edge graphs of trees, edge graphs and traversibility, total graphs, eccentricity sequences and sets, Lesniak theorem for trees, construction of trees, neighbourhoods, Lesniak theorem graphs.

## CREDIT-IV

## Groups in graphs and Graph networks

Automorphism groups of graphs, graph with a given group, Frucht's theorem, Cayley digraph, Transport networks, maximum flow cut and its capacity, Max-flow-Min-cut theorem, multiple sources and sinks, networks containing undirected edges, lower bounds on edge flows, topological sorting, critical path, graphs in game theory, kernel of a digraph

## References

1. R. Balakrishnan, K. Ranganathan, A Text Book of Graph Theory, Springer-Verlag, New York
2. B. Bollobas, Extremal Graph Theory, Academic Press,
3. F. Harary, Graph Theory, Addison-Wesley
4. Narsingh Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall
5. S. Pirzada, An Introduction to Graph Theory, Universities Press, OrientBlackswan, 2012
6. W. T. Tutte, Graph Theory, Cambridge University Press
7. D. B. West, Introduction to Graph Theory, Prentice Hall

Course No. MM14306EA
Course Name:- Mathematical Biology
Duration of Examination: 2:30 Hrs.
No. of Credits: 04

Max. Marks:

Internal Assessment: 20

## CREDIT-I

Diffusion in biology: Fick's law of diffusion, Fick's perfusion law, membrane transport, diffusion through a slab, convective transport, Trancapillary exchange; Heat transport in biological tissues, Oxygen transport through red cells, Gas exchange in lungs, the ideal gas law and solubility of gases, the equation of gas transport in one Alveolus.

## CREDIT-II

Biofluid mechanics: Introduction, various types of fluid flows, viscosity, basic equation of fluid, mechanics, continuity equation, equation of motion, the circulatory system, systemic and pulmonary circulation, the circulation in heart, blood composition, arteries and arterioles, models in blood flow, Poiseulle's flow and its applications, the pulse wave.

## CREDIT-III

Tracers in physiological systems: Compartment systems, the one compartment system, Discrete and continuous transfers, ecomatrix, the continuous infusion, the two compartment system, Bath-tub models, three-compartment system, the leaky compartment and the closed systems, elementary pharmacokinetics, parameter estimation in two compartment models, basic introduction to n compartment systems.

## CREDIT-IV

Biochemical reactions and Population Genetics: The law of mass action, enzyme kinetics, Michael's- Menten theory, Competitive inhibition, Allosteric inhibition, enzyme-substrate-inhibitor system, cooperative properties of enzymes, the cooperative dimer, haemoglobin. Haploid and Diploid genetics, spread of favourite allele, mutation-selection balance, heterosis, frequency dependent selection.

## Books Recommended

1. J.D. Murray, Mathematical Biology, CRC Press
2. S.I. Rubinow, Introduction to Mathematical Biology, John Wiley and Sons
3. Guyton and Hall, Medical Physiology.
4. S.C. Hoppersteadt and C.S. Peskin, Mathematics in Medicine and Life Sciences, Springer-Verlag
5. J.R. Chesnov, Lecture notes in Mathematical Biology, Hong Kong Press
6. J. N. Kapur, Mathematical methods in Biology and Medicine, New Age Publishers
7. D. Ingram and R.F. Bloch, Mathematical methods in Medicine, John Wiley and Sons.

Course No: MM14307EA
Course Name:-Wavelet Theory
Duration of Examination: 2:30 Hrs.
No. of Credits: 04

Max. Marks:
100

External Exam: 80
Internal Assessment: 20

## CREDIT-I

## Time Frequency Analysis and Wavelet Transforms

Gabor transforms basic properties of Gabor transforms, continuous and discrete wavelet transforms with examples, basic properties of wavelet transforms, examples of Haar wavelet, Mexican hat wavelet and their Fourier transforms, dyadic orthonormal wavelet bases for $L^{2}(\mathbb{R})$.

## CREDIT-II

## Multiresolution Analysis and Construction of Wavelets

Definition and examples of multiresolution analysis (MRA), properties of scaling functions and orthonormal wavelet bases, construction of orthonormal wavelets with special reference to Haar wavelet, Franklin wavelet and BattleLemarie wavelet, Spline wavelets, construction of compactly supported wavelets, Daubechie's wavelets and algorithms.

## CREDIT-III

## Other Wavelet Constructions and Characterizations

Introduction to basic equations, some applications of basic equations, characterization of MRA wavelets and scaling functions, construction of biorthogonal wavelets, wavelet packets, definition and examples of wavelets in higher dimensions.

## CREDIT-IV

## Further Extensions of Multiresolution Analysis

Periodic multiresolution analysis and the construction of periodic wavelets, multiresolution analysis associated with integer dilation factor (M-band wavelets), harmonic wavelets, properties of harmonic scaling functions.

## Books Recommended

## Text Book:

1. L. Debnath, Wavelet Transforms and their Applications, Birkhauser, 2002.
2. I. Daubechies, Ten Lectures on Wavelets, CBS-NSF Regional Conferences in Applied Mathematics, 61, SIAM, Philadelphia, PA, 1992.
3. K. Ahmad and F. A. Shah, Introduction to Wavelets with Applications, Real World Education Publishers, New Delhi, 2013.

## Reference Books:

1. C. K. Chui, An Introduction to Wavelets, Academic Press, New York, 1992.
2. M. Pinsky, Introduction to Fourier analysis and Wavelets, Brooks/Cole, 2002.
3. E. Hernandez and G. Weiss, A First Course on Wavelets, CRC Press, New York (1996).

Course No. MM14308EO
Course Name:- Open Elective
Duration of Examination: 2:30 Hrs.
Max. Marks:
100

No. of Credits: 04
External Exam: 80
Internal Assessment: 20

