M.A/M.Sc Mathematics Semester 2nd

Effective from academic session 2010 _____ Repetition for 2012 with minor change

REAL ANALYSIS-II

Course No. MM-CP-202

Unit I

Measure theory: Definition of outer measure and its basic properties, Outer measure of an interval as its length. Countable additivity of the outer measure, Borel measurable sets and Lebesgue measurability of Borel sets, Cantor set, existence of non- measurable sets and of measurable sets which are not Borel, Outer measure of monotonic sequences of sets.

UNIT-II

Measurable functions and their characterization. Algebra of measurable functions, Stienhauss theorem on sets of positive measure, Ostrovisk's theorem on measurable solution of f(x+y)=f(x) + f(y), x, y CR. Convergence a.e., convergence in measure and almost uniform convergenc, their relationship on sets of finite measure, Egoroff's theorem.

UNIT-III

Lebesgue integral of a bounded function. Equivalence of L-integrability and measurability for bounded functions, Riemann integral as a Lebesgue integral, Basic properties of Lebesgue – integral of a bounded function. Fundamental theorem of calculus for bounded derivatives. Necessary and sufficient condition for Riemann integrability on [a, b]. L- integral of non-negative measurable functions and their basic properties. Fatou's lemma and monotone convergence theorem. L–integral of an arbitrary measurable function and basic properties. Dominated convergence theorem and its applications.

UNIT-IV

Absolute continuity and bounded variation, their relationships and counter examples. Indefinite integral of a L-integrable functions and its absolute continuity. Necessary and sufficient condition for bounded variation. Vitali's covering lemma and a.e. differentiability of a monotone function f and $\int f' \leq f(b)-f(a)$.

Recommended Books:

- 1. Royden, L. :Real Analysis (PHI)
- 2. Goldberg, R. : Methods of Real Analysis
- 3. Barra, De. G. : Measure theory and Integration (Narosa)
- 4. Rana ,I.K. : An Introduction to Measure and Integration.
- 5. Rudin, W. Principles of Mathematical Analysis.
- 6. Chae, Lebesgue Integration.
- 7. T.M.Apostol : Mathematical Analysis
- 8. S.M.Shah and Saxena : Real Analysis