# M.A/M.Sc Mathematics Semester 3rd

Effective from academic session 2011 \_\_\_\_\_ Repetition for 2012 with minor change

# ABSTRACT MEASURE THEORY

#### Course No. MM-OP-304

#### Unit-I

Semiring, algebra and  $\sigma$ -algebra of sets, Borel sets, measures on semirings, outer measure associated with a set function and basic properties, measurable sets associated with an outer measure as a  $\sigma$ -algebra, construction of the Lebesgue measure on  $\mathbb{R}^n$ .

## Unit-II

For  $f \in L_1$  [a, b], F'=f a.e. on [a, b]. If f is absolutely continuous on (a, b) with f(x)=0 a.e, then f = constant. Characterization of an absolutely continuous function as an indefinite Lebesgue integral. Non-Lebesgue integrability of f where  $f(x) = x^2 \sin(1/x^2)$ , f(0) = 0 on [0, 1]. Fundamental theorem of calculus for the Lebesgue integral. A brief introduction to  $L_p$  spaces. Holder's and Minkowki's inequalities.

## **Unit-III**

Improper Riemann integral as a Lebesgue integral, calculation of some improper Riemann integrable functions, space of Lebesgue integrable functions as completion of Riemann integrable functions on [a,b], change of variables formula and simple consequences, Riemann Lebesgue lemma.

## **Unit-IV**

Product measures and iterated integrals, example of non-integrable functions whose iterated integrals exist (and are equal), Fubini theorem, expressing a double integral as an iterated integral, Tonelli-Hobson theorem as a converse to Fubini theorem, differentiation under the integral sign.

## **Recommended Books:**

1.C.D.Aliprantis and O.Burkinshaw, Principles of Real Analysis

2.Goldberg, R. : Methods of Real Analysis

3.T.M.Apostol : Mathematical Analysis

## Suggested Readings:

1.Royden, L: Real Analysis (PHI)

- 2. Chae, S.B. Lebesgue Integration(Springer Verlag).
- 3. Rudin, W. Principles of Mathematicals Analysis (McGraw Hill).

4.Barra ,De. G. : Measure theory and Integration (Narosa)

5.Rana ,I.K. : An Introduction to Measure and Integration, Narosa Publications.