POST GRADUATE DEPARTMENT OF MATHEMATICS

UNIVERSITY OF KASHMIR, SRINAGAR - 190006

Course Structure for 2015 onwards (CBCS Advanced)

Professor & Head
Postgraduate Department
of Mathematics
University of Kashmir
Srinagar

Semester – I				
Course Type	**Course Code**	**Title of the Course**	**No. of Credits**	**Teacher**
Core (CR)	MM15101CR	Advanced Abstract Algebra – I	04	
	MM 15102CR	Real Analysis – I	04	
	MM 15103CR	Topology	04	
Discipline Centric Electives (DCE)	MM 15104DCE	Theory of Matrices	04	
	MM 15105DCE	Theory of Numbers-I	02	
	MM 15106DCE	Advanced Calculus	02	
	MM 15107DCE	Probability Theory	02	
Generic Electives (GE)	MM 15108GE	Introduction to Real Analysis	02	
	MM 15109GE	Numerical Methods	02	
Open Electives (OE)	MM 15110OE	Calculus	02	
	MM 15111OE	Introduction to Numbers	02	

General Instructions for the Candidates

1. The two year (4 semester) PG programmes is of 96 credit weightage i.e., 24 credits/semester (24x4=96).
2. Out of 24 credits in a semester a candidate has to obtain 12 credits compulsorily from "**Core Courses**", while the remaining 12 credits can be obtained from the "**Electives**" in the following manner:
   - A candidate can obtain a maximum of 6 credits within his/her own Department out of the specializations offered by the Department as **Discipline Centric-Electives**.
   - 6 credits shall be obtained by a candidate from the **Electives** offered by the Department other than his/her own. The candidate shall be free to obtain these 6 credits from the **Generic** or **Open Electives** or **a combination of both**.

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SEMESTER-I

ADVANCED ABSTRACT ALGEBRA-I

Course No: **MM15101CR**	Max. Marks: 100
Duration of Examination: 2:30 Hrs.	External Exam: 80
No. of Credits: **04**	Internal Assessment: 20

CREDIT-I

Definitions and examples of semi-groups and monoids, criteria for the semi-groups to be a group, cyclic groups, structure theorem for cyclic groups, endomorphism, automorphism, inner automorphism and outer automorphism, center of a group, Cauchy's and Sylow's theorem for abelian groups, permutation groups, symmetric groups, alternating groups, simple groups, simplicity of the alternating group A_n for $n \geq 5$.

CREDIT-II

Normalizer, conjugate classes, class equation of a finite group and its applications, Cauchy's and Sylow's theorems for finite groups, double cosets, second and third parts of Sylow's theorem, direct product of groups, finite abelian groups, normal and subnormal series, composition series, Jordan Holder theorem for finite groups, Zassenhaus lemma, Schreir's refinement theorem, Solvable groups.

CREDIT-III

Brief review of rings, integral domain, ideals, the field of quotients of an integral domain, embedding of an Integral domain, Euclidean rings with examples such as $Z[\sqrt{-1}]$, $Z[\sqrt{2}]$, principal ideal rings(PIR), unique factorization domains(UFD) and Euclidean domains, greatest common divisor, lowest common multiple in rings, relationships between Euclidean rings, P.I.R.'s and U.F.D.

CREDIT-IV

Polynomial rings, the division algorithm for polynomials, irreducible polynomials, polynomials and the rational field, primitive polynomials, contraction of polynomials, Gauss lemma, Integer monic polynomial, Eisenstein's irreducibility criterion, cyclotomic polynomials, polynomial rings and commutative rings.

Recommended Books

1. P. B. Bhatacharaya and S.K.Jain, Basic Abstract Algebra.
2. J. B. Fragleigh, A First Course in Abstract Algebra.
3. J. A. Gallian, Contemporary Abstract Algebra.
4. I. N. Heristein, Topics in Algebra.
5. K. S. Miller, Elements of Modern Abstract Algebra.
6. Surjeet Singh and Qazi Zameer-ud-Din, Modern Algebra, Vikas Publishing House Private Limited.

REAL ANALYSIS - I

Course No: **MM 15102CR**	Max. Marks: 100
Duration of Examination: 2:30 Hrs.	External Exam: 80
No. of Credits: **04**	Internal Assessment: 20

CREDIT-I

Integration : Definition and existence of Riemann – Stieltje's integral , behavior of upper and lower sums under refinement, necessary and sufficient conditions for RS-integrability of continuous and monotonic functions, reduction of an RS-integral to a Riemann integral, basic properties of RS-integrals, differentiability of an indefinite integral of continuous functions, the fundamental theorem of calculus for Riemann integrals.

CREDIT-II

Improper integrals: integration of unbounded functions with finite limit of integration, comparison tests for convergence, Cauchy's test, infinite range of integration, absolute convergence, integrand as a product of functions, Abel's and Dirichlet's test.

Inequalities: arithmetic-geometric means equality, inequalities of Cauchy Schwartz, Jensen, Holder & Minkowski, inequality on the product of arithmetic means of two sets of positive numbers.

CREDIT-III

Infinite series: Carleman's theorem, conditional and absolute convergence, multiplication of series, Merten's theorem, Dirichlet's theorem, Riemann's rearrangement theorem. Young's form of Taylor's theorem, generalized second derivative, Bernstein's theorem and Abel's limit theorem.

CREDIT-IV

Sequences and series of functions: point wise and uniform convergence, Cauchy criterion for uniform convergence, M_n-test, Weiestrass M-test, Abel's and Dirichlet's test for uniform convergence, uniform convergence and continuity, R- integration and differentiation, Weiestrass approximation theorem, example of continuous nowhere differentiable functions.

Recommended Books:

1. R. Goldberg, Methods of Real Analysis.
2. W. Rudin, Principles of Mathematical Analysis.
3. J. M. Apostol, Mathematical Analysis.
4. S.M.Shah and Saxen, Real Analysis.
5. A.J.White, Real Analysis , An Introduction.
6. L.Royden, Real Analysis.
7. S.C.Malik and Gupta, Real Analysis.

TOPOLOGY

Course No: **MM 15103CR**
Duration of Examination: 2:30 Hrs.
No. of Credits: **04**

Max. Marks:	100
External Exam:	80
Internal Assessment:	20

CREDIT-I

Review of countable and uncountable sets, Schroeder-Bernstein theorem, axiom of choice and its various equivalent forms, definition and examples of metric spaces, open and closed sets, completeness in metric spaces, Baire's category theorem, and applications to the (1) non-existence of a function which is continuous precisely at irrationals (ii) impossibility of approximating the characteristic of rationals on [0, 1] by a sequence of continuous functions.

CREDIT-II

Completion of a metric space, Cantor's intersection theorem with examples to demonstrate that each of the conditions in the theorem is essential, uniformly continuous mappings with examples and counter examples, extending uniformity continuous maps, Banach's contraction principle with applications to the inverse function theorem in R.

CREDIT-III

Topological spaces; definition and examples, elementary properties, Kuratowski's axioms, continuous mappings and their characterizations, pasting lemma, convergence of nets and continuity in terms of nets, bases and sub bases for a topology, lower limit topology, concepts of first countability, second countability, separability and their relationships, counter examples and behavior under subspaces, product topology and weak topology, compactness and its various characterizations.

CREDIT-IV

Heine-Borel theorem, Tychnoff's theorem, compactness, sequential compactness and total boundedness in metric spaces, Lebesgue's covering lemma, continuous maps on a compact space, separation axioms T_i $\left(i=1,2,3,3\frac{1}{2},4\right)$ and their permanence properties, connectedness, local connectedness, their relationship and basic properties, connected sets in R, Urysohn's lemma, Urysohn's metrization theorem, Tietize's extension theorem, one point compactification.

Recommended Books:

1. G.F.Simmons, Introduction to Topology and Modern Analysis.
2. J. Munkres, Topology.
3. K.D. Joshi, Introduction to General Topology.
4. J.L.Kelley, General Topology.
5. Murdeshwar, General Topology.
6. S.T. Hu, Introduction to General Topology.

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THEORY OF MATRICES

Course No. **MM 15104DCE**		Max. Marks:	100
Duration of Examination: 2:30 Hrs.		External Exam:	80
No. of Credits: **04**		Internal Assessment:	20

UNIT-I

Eigen values and eigen vectors of a matrix and their determination, similarity of matrices, two similar matrices have the same eigen values, algebraic and geometric multiplicity, necessary and sufficient condition for a square matrix of order n to be similar to a diagonal matrix, orthogonal reduction of real matrices.

UNIT II

Orthogonality of the eigen vectors of a Hermitian matrix, the necessary and sufficient condition for a square matrix of order n to be similar to a diagonal matrix . If A is a real symmetric matrix then there exists an orthogonal matrix P such that $P^{-1}AP = P`AP$ is a diagonal matrix whose diagonal elements are the eigen values of A, semi–diagonal or triangular form, Schur's theorem, normal matrices, necessary and sufficient condition for a square matrix to be unitarily similar to a diagonal matrix.

UNIT-III

Quadratic forms: the Kroneckers and Lagranges reduction, reduction by orthogonal transformation of real quadratic forms, necessary and sufficient condition for a quadratic form to be positive definite, rank, index and signature of a quadratic form. If $A=[a_{ij}]$ is a positive definite matrix of order n, then $|A| \leq a_{11}a_{22}\ldots a_{nn}$.

UNIT IV

Gram matrices: the Gram matrix BB` is always positive definite or positive semi-definite, Hadamard's inequality, If $B=[b_{ij}]$is an arbitrary non- singular real square matrix of order n, then $|B| \leq \prod_{i=1}^{n}[\sum_{k=1}^{n} b_{ik}]$, functions of symmetric matrices, positive definite square root of a positive definite matrix, the infinite n-fold integral

$$I_n = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} e^{-X'AX}\,dX\,,$$

where $dX = dx_1 dx_2 \cdots dx_n$. If A is a positive definite matrix , then $I_n = \dfrac{\pi^{n/2}}{|A|^{1/2}}$

If A and B are positive definite matrices, then $|\lambda A + (1-\lambda)B| \geq |A|^{\lambda}|B|^{1-\lambda}$ for $0 \leq \lambda \leq 1$,

perturbation of roots of polynomials, companion matrix, Hadamard's theorem, Gerishgorian Disk theorem, Taussky's theorem.

<u>Recommended Books:</u>

1 Richard Bellman, Introduction to Matrix Analysis, McGraw Hill Book Company.
2 Franz E. Hohn, Elementary Matrix Algebra, American Publishing company Pvt. Ltd.
3 Shanti Narayan, A Text Book of Matrices, S. Chand and Company Ltd.
4 Rajendra Bhatia, Matrix Analysis, Springer.

THEORY OF NUMBERS-I

Course No. **MM 15105DCE**	Max. Marks: 50
Duration of Examination: 1:30 Hrs.	External Exam: 40
No. of Credits: **02**	Internal Assessment: 10

CREDIT-I

Division algorithm, greatest common divisor and its properties, the Euclidean algorithm, prime numbers, Euclid's first theorem, fundamental theorem of arithmetic, divisor of n, Radix-representation, linear diophantine equations, sequence of primes, Euclid's second theorem, infinitude of primes of the form 4n+3 and of the form 6n+5, congruences, complete residue system (CRS), reduced residue system (RRS) and their properties, Fermat and Euler's theorems with applications, Euler's ϕ-function, $\phi(mn) = \phi(m)\,\phi(n)$ where $(m, n) = 1$, Wilson's theorem and its application to the solution the congruence of $x^2 \equiv -1 \pmod p$.

CREDIT-II

Solutions of linear congruence's, Chinese Remainder theorem, congruences of higher degree $F(x) \equiv 0 \pmod m$, where $F(x)$ is a polynomial, congruences with prime power, congruences with prime modulus and related results, Lagrange's theorem, factor theorem and its generalization, polynomial congruences $F(x_1, x_2 \ldots x_n) \equiv 0 \pmod p$ in several variables, equivalence of polynomials, Chevalley's theorem, Warning's theorem, quadratic forms over a field of characteristic $\neq 2$, equivalence of quadratic forms, Witt's theorem, representation of Field Elements, Hermite's theorem.

Recommended Books:

1. W. J . Leveque, Topics in Number Theory, Vol. I and II Addition Wesley Publishing Company, INC.

2. I. Niven and H.S Zuckerman, An introduction of the Theory of Numbers.

3. Boevich and Shaferivich, Number Theory, I.R, Academic Press.

References:

1. T.M Apostal, Analytic Number Theory, Springer Verlag.
2. G.H Hardy and Wright, An introduction to the theory of Numbers.
3. J.P. Serre, A course in Arithmetic, GTM Vol. Springer Verlag 1973.
4. E. Landau, An Elementary Number Theory.

ADVANCED CALCULUS

Course No. **MM 15106DCE**	Max. Marks: 50
Duration of Examination: 1:30 Hrs.	External Exam: 40
No. of Credits: **02**	Internal Assessment: 10

CREDIT-I

Functions of several variables in R^n, the directional derivative, directional derivative and continuity, total derivative, matrix of a linear function, Jacobian matrix, chain rule, mean value theorem for differentiable functions.

CREDIT-II

Sufficient conditions for differentiability and for the equality of mixed partials, Taylor's theorem for functions from R^n and R, inverse and implicit function theorem in R^n , extremum problems for functions on R^n, Lagrange's multiplier's, multiple Riemann Integral and change of variable formula for multiple Riemann integrals.

Recommended Books:

1. W.Rudin, Principles of Mathematical Analysis.
2. T.M.Apostol, Mathematical Analysis.
3. S.M.Shah and Saxen, Real Analysis.

PROBABILITY THEORY

Course No. **MM 15107DCE**	Max. Marks: 50
Duration of Examination: 1:30 Hrs.	External Exam: 40
No. of Credits: **02**	Internal Assessment: 10

UNIT- I

The probability set functions, its properties, probability density function, the distribution function and its properties, mathematical expectations, some special mathematical expectations, inequalities of Markov, Chebyshev and Jensen.

UNIT-II

Conditional probability, independent events, Baye's theorem, distribution of two and more random variables, marginal and conditional distributions, conditional means and variances, correlation coefficient, stochastic independence and its various criteria.

Recommended Books:

1. Hogg and Craig, An Introduction to the Mathematical Statistics.
2. Mood and Grayball, An Introduction to the Mathematical Statistics.

INTRODUCTION TO REAL ANALYSIS

Course No. **MM 15108GE**	Max. Marks:	50
Duration of Examination: 1:30 Hrs.	External Exam:	40
No. of Credits: **02**	Internal Assessment:	10

CREDIT-I

Review of real numbers, countable and uncountable sets, countability of rationals and uncountablity of reals, bounded and unbounded sets, lub and glb of a set, sequences and their limits, bounded and monotonic sequences, convergent and divergent sequences, limit inferior and limit superior, infinite series and their convergence and divergence, tests of convergence of positive term series.

CREDIT-II

Continuity, differentiability and integrability of real functions, properties of continuous functions in a closed interval, derivatives and integrals of some special functions, continuity, differentiation and integration of sum, difference, product and quotient of functions, Rolle's theoremand its applications, Mean value theorem, expression of a function as a series.

NUMERICAL METHODS

Course No. **MM 15109GE**
Duration of Examination: 1:30 Hrs.
No. of Credits: **02**

Max. Marks:	50
External Exam:	40
Internal Assessment:	10

CREDIT-I

Solution of algebraic and transcendental and polynomial equations, bisection method, iteration method based on first degree equation, secant method, regula-falsi method, Newton-Raphson method, rate of convergence of Newton-Raphson method & secant method, system of linear algebraic equation, Gauss elimination method, Gauss Jordan method.

CREDIT-II

Interpolation and approximation of finite difference operators, Newton's forward, backward interpolation, central difference interpolation, Lagrange's interpolation, Newton Divided Difference interpolation, Hermite interpolation, Spline interpolation, numerical differentiation and Integration.

Recommended Books:

1. M.K. Jain, Numerical solution of differential equations, Wiley Eastern (1979), Second Edition.
2. D.V. Griffiths and I.M. Smith, Numerical Methods for Engineers, Blackwell Scientific Publications (1991).

REFERENCES:

1. S.C. Chapra, and P.C. Raymond, Numerical Methods for Engineers, Tata McGraw Hill, New Delhi (2000)
2. R.L. Burden, and J. Douglas Faires, Numerical Analysis, P.W.S. Kent Publishing Company, Boston (1989), Fourth edition.
3. S.S. Sastry, Introductory methods of Numerical analysis, Prentice- Hall of India, New Delhi (1998).
4. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical methods for scientific and Engineering computation, Wiley Eastern (1993)

CALCULUS

Course No. **MM 15110OE** | Max. Marks: 50
Duration of Examination: 1:30 Hrs. | External Exam: 40
No. of Credits: **02** | Internal Assessment: 10

CREDIT-I

Functions, the idea of limits, techniques for computing limits, infinite limits, continuity, derivative, rules for differentiation, derivatives as rate of change, applications of the derivative, maxima and minima, increasing and decreasing functions, mean value theorem and its applications, indeterminate forms, partial differentiation, Euler's theorem.

CREDIT-II

Indefinite integral, techniques of integration, definite integral, area of a bounded region, applications of integration, velocity and net change, region between curves, volume by slicing, volume by shells, length of curves, physical applications.

Recommended Books:

1. A.Aziz, S.D.Chopra and M.L.Kochar, Differential Calculus, Kapoor Publications.
2. William L.Briggs and Lyle Cochran, Calculus, Pearson.
3. S.D.Chopra and M.L.Kochar, Intgeral Calculus, Kapoor Publications.
4. R.K.Jain and S.R.K. Lyengar, Advanced Engineering Mathematics, Narosa.

INTRODUCTION TO NUMBERS

Course No. **MM 15111OE**		Max. Marks:	50
Duration of Examination: 1:30 Hrs.		External Exam:	40
No. of Credits: **02**		Internal Assessment:	10

CREDIT-I

Number system, basic binary operations, ordering of integers, well ordering principle, principle of mathematical induction, radix representations, divisibility, greatest common divisor (gcd), least common multiple (lcm) and their properties, pigeonhole principle.

CREDIT-I

Prime and composite numbers, relatively prime numbers, infinitude of prime numbers, primes of different forms, perfect numbers, fundamental theorem of arithmetic, congruence's and their properties.

Recommended Books:

1. W.J.Leveque, Topics in Number Theory, Addison Wesley Publishing Company.
2. Ivan Niven & H.S.Zuckerman, An introduction to the Theory of Numbers, Wiley Eastern Ltd.
3. G.H.Hardy & E.M.Wright, An introduction to the Theory of Numbers, Oxford University Press 1954.
4. H.N.Wright, First Course in Theory of Numbers, John Wiley & Sons, New York 1939.