

Semester - IV				
Course Type	Course Code	Title of the Course	No. of Credits	Teacher
Core (CR)	MM17401CR	Partial Differential Equations	04	
	MM17402CR	Differential Geometry	04	
	MM17403CR	Advanced Abstract Algebra-II	04	
Discipline Centric Electives (DCE)	MM17404DCE	Analytic Theory of Polynomials	04	
	MM17405DCE	Mathematical Statistics	04	
	MM17406DCE	Functional Analysis - II	04	
	MM17407DCE	Non-Linear Analysis	04	
	MM17408DCE	Advanced Topics in Topology and Modern Analysis	04	
	MM17409DCE	Latex and Matlab	02	
	MM17410DCE	Project	02	
Generic Electives (GE)	MM17007GE	Applied Group Theory	02	
	MM17008GE	Basic Graph Theory	02	
Open Electives (OE)	MM17004OE	Lattices and Boolean Algebra	02	

#### General Instructions for the Candidates

1. The two year (4 semester) PG programme is of 96 credit weightage i.e., 24 credits/semester (24x4=96).
5. Out of 24 credits in a semester a candidate has to obtain 12 credits compulsorily from "**Core Courses**", while the remaining 12 credits can be obtained from the "**Electives**" in the following manner:
  - A candidate can obtain a maximum of 8 credits within his/her own Department out of the specializations offered by the Department as **Discipline Centric-Electives**.
  - 4 credits shall be obtained by a candidate from the **Electives** offered by the Department other than his/her own. The candidate shall be free to obtain either 4 credits from the **Generic** (within School ) or two credits from Generic (within School) and two credits from **Open Electives**

The Academic Tour shall be conducted by the Department every year for outgoing students (4<sup>th</sup> semester).



## **SEMESTER-IV**

### **PARTIAL DIFFERENTIAL EQUATIONS**

Course No: <b>MM 17401CR</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
<u>End Term Exam: (2 Credits)</u>	<u>Max.Marks:</u>	<u>50</u>

#### **CREDIT-I**

Introduction to partial differential equations, partial differential equations of first order, linear and non-linear partial differential equations, Lagrange's method for the solution of linear partial differential equations, Charpits method and Jacobi methods for the solution of non-linear partial differential equations, initial-value problems for quasi-linear first-order equations, Cauchy's method of characteristics.

#### **CREDIT-II**

Origin of second order partial differential equations, linear partial differential equations with constant coefficients, methods for solution of second order partial differential equations, classification of second order partial differential equations, canonical forms, adjoint operators, Riemann's method, Monge's method for the solution of non-linear partial differential equations.

#### **CREDIT-III**

Derivation of Laplace and heat equations, boundary value problems, Dirichlet's and Neumann problems for a circle and sphere; solutions by separation of variables method, cylindrical coordinates and spherical polar coordinate system, maximum-minimum principle, uniqueness theorem, Sturm-Liouville theory.

#### **CREDIT-IV**

Derivation of wave equation, D' Alembert's solution of one dimensional wave equation, separation of variables method, periodic solutions; method of eigen functions, Duhamel's principle for wave equation, Laplace and Fourier transforms and their applications to partial differential equations, Green function method and its applications.



### **Recommended Books:**

1. Robert C. McOwen, Partial Differential Equations-Methods and Applications, Pearson Education, Delhi, 2004.
2. L. C. Evans, Partial Differential Equations, GTM, AMS, 1998
3. Diran Basmadjian, The Art of Modelling in Science and Engineering, Chapman & Hall/CRC, 1999.
4. E. DiBenedetto, Partial Differential Equations, Birkhauser, Boston, 1995.
5. F. John, Partial Differential Equations, 3<sup>rd</sup> ed., Narosa Publ. Co., New Delhi, 1979.
6. E. Zauderer, Partial Differential Equations of Applied Mathematics, 2<sup>nd</sup> ed., John Wiley and Sons, New York, 1989



## DIFFERENTIAL GEOMETRY

Course No: <b>MM 17402CR</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
<u>End Term Exam: (2 Credits)</u>	<u>Max.Marks:</u>	<u>50</u>

### CREDIT-I

Curves: differentiable curves, regular point, parameterization of curves, arc-length, arc-length is independent of parameterization, unit speed curves, plane curves, curvature of plane curves, osculating circle, centre of curvature. computation of curvature of plane curves, directed curvature, examples, straight line, circle, ellipse, tractrix, evolutes and involutes, space curves, tangent vector, unit normal vector and unit binormal vector to a space curve, curvature and torsion of a space curve, the Frenet-Serret theorem, first fundamental theorem of space curves, intrinsic equation of a curve, computation of curvature and torsion, characterization of helices and curves on sphere in terms of their curvature and torsion, evolutes and involutes of space curves.

### CREDIT-II

Surfaces: regular surfaces with examples, coordinate charts or curvilinear coordinates, change of coordinates, tangent plane at a regular point, normal to the surface, orientable surface, differentiable mapping between regular surfaces and their differential, fundamental form or a metric of a surface, line element, invariance of a line element under change of coordinates, angle between two curves, condition of orthogonality of coordinate curves, area of bounded region, invariance of area under change of coordinates.

### CREDIT-III

Curvature of a Surface: normal curvature, Euler's work on principal curvature, qualitative behavior of a surface near a point with prescribed principal curvatures, the Gauss map and its differential, the differential of Gauss is self-adjoint, second fundamental form, normal curvature in terms of second fundamental form. Meunier theorem, Gaussian curvature, Weingarten equation, Gaussian curvature  $K(p) = (eg-f^2)/EG-F^2$ , surface of revolution, surfaces with constant positive or negative Gaussian curvature, Gaussian curvature in terms of area, line of curvature, Rodrigue's formula for line of



curvature, equivalence of surfaces, isometry between surfaces, local isometry, and characterization of local isometry.

#### **CREDIT-IV**

Christoffel symbols, expressing Christoffel symbols in terms of metric coefficients and their derivative, Theorema egregium (Gaussian curvature is intrinsic), isometric surfaces have same Gaussian curvatures at corresponding points, Gauss equations and Manardi Codazzi equations for surfaces, fundamental theorem for regular surface. (Statement only).

Geodesics: geodesic curvature, geodesic curvature is intrinsic, equations of geodesic, geodesic on sphere and pseudo sphere, geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement only), geodesic triangle on sphere, implication of Gauss-Bonnet theorem for geodesic triangle.

#### **Recommended Books:**

1. John Mc Cleary, Geometry from a differentiable Viewpoint. (Cambridge Univ. Press).
2. W. Klingenberg, A course in Differential Geometry (Spring Verlag).
3. C. E. Weatherburn, Differential Geometry of Three dimensions.
4. T. Willmore, An Introduction to Differential Geometry.
5. J. M. Lee, Riemannian Manifolds, An Introduction to Curvature.



## ADVANCED ABSTRACT ALGEBRA-II

Course No: <b>MM 17403CR</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
End Term Exam: (2 Credits)	Max.Marks:	50

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### **CREDIT-I**

Relation and ordering, partially ordered sets, lattices, properties of lattices, lattices as algebraic systems, sub-lattices, direct product and homomorphism, modular lattices, complete lattices, bounds of lattices, distributive Lattice, complemented lattices.

### **CREDIT-II**

Modules, sub-modules, quotient modules, homomorphism and isomorphism theorem, cyclic modules, simple modules, semi-simple modules, Schuer's lemma, free modules, ascending chain condition and maximum condition, and their equivalence, descending chain condition and minimum condition and their equivalence, direct sums of modules, finitely generated modules.

### **CREDIT-III**

Fields: Prime fields and their structure, extensions of fields, algebraic numbers and algebraic extensions of a field, roots of polynomials, remainder and factor theorems, splitting field of a polynomial, existence and uniqueness of splitting fields of polynomials, simple extension of a field.

### **CREDIT-IV**

Separable and in-separable extensions, the primitive element theorem, finite fields, perfect fields, the elements of Galois theory, automorphisms of fields, normal extensions, fundamental theorem of Galois theory, construction with straight edge and compass,  $\mathbb{R}^n$  is a field iff  $n = 1, 2$ .

### **Recommended Books:**

1. I. N. Heristein, Topics in Algebra.
2. K. S. Miller, Elements of Modern Abstract Algebra.
3. Surjeet Singh and Qazi Zameer-ud-din, Modern Algebra, Vikas Publishers Pvt. Limited.



## ANALYTIC THEORY OF POLYNOMIALS

Course No: <b>MM 17404DCE</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
<u>End Term Exam: (2 Credits)</u>	<u>Max.Marks:</u>	<u>50</u>

### **CREDIT-I**

Introduction, the fundamental theorem of algebra(revisited), symmetric polynomials, the continuity theorem, orthogonal polynomials, general properties, the classical orthogonal polynomials, tools from matrix analysis.

### **CREDIT-II**

Critical points in terms of zeros, fundamental results and critical points, convex hulls and Gauss-Lucas theorem, some applications of Gauss-Lucas theorem, extensions of Gauss-Lucas theorem, average distance from a line or a point, real polynomials and Jensen's theorem, extensions of Jensen's theorem.

### **CREDIT-III**

Derivative estimates on the unit interval, inequalities of S. Bernstein and A. Markov, extensions of higher order derivatives, two other extensions, dependence of the bounds on the zeros, some special classes,  $L^p$  analogous of Markov's inequality.

### **CREDIT-IV**

Coefficient estimates, polynomials on the unit circles, coefficients of real trigonometric polynomials, polynomials on the unit interval.

### **Recommended Books:**

1. Q. I. Rahman and G.Schmeisser, Analytic Theory of Polynomials.
2. Morris Marden, Geometry of Polynomials.
3. G. V. Milovanovic, D.S.Mitrinovic and Th. M. Rassias, Topics in Polynomials, Extremal Properties, Problems, Inequalities, Zeroes.
4. G. Polya and G. Szego, Problems and Theorems in Analysis ( Springer Verlag New York Heidelberg Berlin).



## MATHEMATICAL STATISTICS

Course No: <b>MM 17405DCE</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
End Term Exam: (2 Credits)	Max.Marks:	<u>50</u>

### **CREDIT-I**

Some Special Distributions, Bernoulli, Binomial, trinomial, multinomial, negative binomial, Poisson, gamma, chi-square, beta, Cauchy, exponential, geometric, normal and bivariate normal distributions.

### **CREDIT-II**

Distribution of functions of random variables, distribution function method, change of variables method, moment generating function method, t and F distributions, Dirichelet distribution, distribution of order statistics, distribution of  $X$  and  $\frac{nS^2}{\sigma^2}$ , limiting distributions, different modes of convergence, central limit

### **CREDIT-III**

Interval estimation, confidence interval for mean, confidence interval for variance, confidence interval for difference of means and confidence interval for the ratio of variances, point estimation, sufficient statistics, Fisher-Neyman criterion, factorization theorem, Rao- Blackwell theorem, best statistic (MvUE), Complete Sufficient Statistic, exponential class of pdfs.

### **CREDIT-IV**

Rao-Crammer inequality, efficient and consistent estimators, maximum likelihood estimators (MLE's), testing of hypotheses, definitions and examples, best or most powerful (MP) tests, Neyman Pearson theorem, uniformly most powerful (UMP) tests, likelihood ratio test, chi-square test.



### **Recommended Books**

1. Hogg and Craig, An Introduction to Mathematical Statistics.
2. Mood and Graybill, An Introduction to Mathematical Statistics.

### **References**

1. C. R. Rao, Linear Statistical Inference and its Applications.
2. V. K. Rohatgi, An Introduction to Probability and Statistics.



## FUNCTIONAL ANALYSIS-II

Course No: <b>MM 17406DCE</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
<u>End Term Exam: (2 Credits)</u>	Max.Marks:	50

### **CREDIT-I**

Relationship between analytic and geometric forms of Hahn-Banach theorem, applications of Hahn-Banach theorem, Banach limits, Markov-Kakutani theorem for a commuting family of maps, complemented subspaces of Banach spaces, complementability of dual of a Banach space in its bidual, uncomplementability of  $c_0$  in  $l_\infty$ .

### **CREDIT-II**

Dual of subspaces, quotient spaces of a normed linear space, weak and weak\* topologies on a Banach space, Goldstine's theorem, Banach Alaoglu theorem and its simple consequences, Banach's closed range theorem, injective and surjective bounded linear mappings between Banach spaces.

### **CREDIT-III**

$l_\infty$  and  $C[0,1]$  as universal separable Banach spaces,  $l_1$  as quotient universal separable Banach spaces, Reflexivity of Banach spaces and weak compactness, Completeness of  $L_p[a,b]$ , extreme points, Krein-Milman theorem and its simple consequences.

### **CREDIT-IV**

Dual of  $l_\infty$ ,  $C(X)$  and  $L_p$  spaces. Mazur-Ulam theorem on isometries between real normed spaces, Muntz theorem in  $C[a,b]$ .

### **Recommended Books:**

1. J. B. Conway, A First Course in Functional Analysis (Springer Verlag).
2. R. E. Megginson, An Introduction to Banach Space theory (Springer Verlag, GTM, Vol. 183)
3. Lawrence Baggett, Functional Analysis, A Primer (Chapman and Hall, 1991).

### **References:**

1. B. Ballobas, Linear Analysis (Camb. Univ.Pres).
2. B. Beauzamy, Introduction to Banach Spaces and their geometry ( North Holland ).
3. Walter Rudin, Functional Analysis (Tata McGrawHill).



## NON-LINEAR ANALYSIS

Course No: <b>MM 17407DCE</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
End Term Exam: (2 Credits)	Max.Marks:	50

### **CREDIT-I**

Convex Sets, best approximation properties, topological properties, separation, nonexpansive operators, projectors onto convex sets, fixed points of nonexpansive operators, averaged nonexpansive operators, Fejer monotone sequences, convex cones, generalized interiors, polar and dual cones, tangent and normal cones, convex functions, variants, between linearity and convexity, uniform and strong convexity, quasiconvexity

### **CREDIT-II**

Gateaux Derivative, Frechet Derivative, lower semicontinuous convex functions, subdifferential of convex functions, directional derivatives, characterization of convexity and strict convexity, directional derivatives and subgradients, Gateaux and Frechet differentiability, differentiability and continuity

### **CREDIT-III**

Monotone operators, strong notions of monotonicity such as para, cyclic, strict, uniform and strong monotonicity, maximal monotone operator and their properties, bivariate functions and maximal monotonicity, Debrunner-Flor theorem, Minty theorem, Rockfeller's cyclic monotonicity theorem, monotone operators on  $R$ .

### **CREDIT-III**

Reisz-Representation theorem, projection mappings and their properties, characterization of projection onto convex sets and their geometrical interpretation,

Billinear forms and its applications, Lax-Milgram lemma, minimization of functionals, variational inequalities, relationship between abstract



minimization problems and variational inequalities, Lions Stampacchia theorem for existence of solution of variational inequality.

**Recommended Books:**

1. H. H. Bauschke and P. L. Combettes, Convex Analysis and Monotone Operator Theory in Hilbert Spaces, Springer New York, 2011.
2. D. Kinderlehrer and G. Stampacchia, An Introduction to Variational Inequalities and Their Applications, Academic Press, New York, 1980.
3. A. H. Siddiqi, K. Ahmed and Manchanda, P. Introduction to Functional Analysis with Applications, Anamaya Publishers, New Delhi-2006.

**References:**

1. I. Ekeland and R. Temam, Convex Analysis and Variational Problems, W.Takahashi, Nonlinear Functional Analysis, North-Holland Publishing Company-Ammsterdam, 1976.
2. M. C. Joshi and R. K. Bose, Nonlinear Functional Analysis and its Applications, Willey Eastern Limited, 1985.



## **ADVANCED TOPICS IN TOPOLOGY AND MODERN ANALYSIS**

Course No: <b>MM 17408DCE</b>	Total Credits:	<b>04</b>
Continuous Exam-I (1 Credit)	Max.Marks:	25
Continuous Exam-II (1 Credit)	Max.Marks:	25
End Term Exam: (2 Credits)	Max.Marks:	50

### **CREDIT-I**

Uniform spaces, definition and examples, uniform topology, metrizable complete regularity of uniform spaces, pre-compactness and compactness in uniform spaces, uniform continuity.

### **CREDIT-II**

Uniform continuity, uniform continuous maps on compact spaces Cauchy convergence and completeness in uniform spaces, initial uniformity, simple applications to function spaces, Arzela- Ascoli theorem.

### **CREDIT-III**

Abstract harmonic analysis, definition of a topological group and its basic properties. subgroups and quotient groups, product groups and projective limits, properties of topological groups involving connectedness, invariant metrics and Kakutani theorem, structure theory for compact and locally compact, Abelian groups.

### **CREDIT-IV**

Some special theory for compact and locally compact Abelian groups, Haar integral and Haar measure, invariant means defined for all bounded functions, convolution of functions and measures, elements of representation theory, unitary representations of locally compact groups.

### **Recommended Books:**

1. I. M. James, Uniform Spaces, Springer Verlag.
2. K. D. Joshi, Introduction to General Topology.
3. S. K. Berberian, Lectures on Operator Theory and Functional Analysis, Springer Verlag.
4. G. B. Folland, Real Analysis, John Wiley.

### **References:**

1. G. Murdeshwar, General Topology.
2. E. Hewitt & K.A Ross, Abstract Harmonic Analysis-I, Springer Verlag.



## LATEX AND MATLAB

Course No: <b>MM 17409DCE</b>	Total Credits:	<b>02</b>
End Term Exam: (2 Credits)	Max.Marks:	25
	Max.Marks:	25

### **CREDIT-I**

#### **LateX**

Purpose and nature of LateX, LateX workflow, LateX philosophy and user interface, advantages over word processors, basic text formatting, equation formatting, introduction to TeX programming, insertion and deletion of mathematical formulae in LateX, formation of graphics in LateX, creating power point of LateX file.

### **CREDIT-II**

#### **MATLAB**

Introduction to MATLAB, basic features, array and array operations: simple array, array construction and orientation, array mathematics, standard arrays, manipulation and sorting, multidimensional arrays: array construction, array mathematics and manipulation, relational and logical operations, control flow, functions, matrix algebra: sets of linear equations, matrix functions, special matrices, data analysis and statistical functions, polynomials: roots, multiplications, addition, division, derivatives and integral evaluation.

#### **Recommended Books:**

1. Helmut and Partik W.Daly, Guide to LateX.
2. M.Goossens, F.Mittelbach, S.Rahtz, D.Roegel and H.Voss, The Latext Graphics Companion, 2<sup>nd</sup> Edition.
3. Duane Hanselman, Mastering MATLAB, Bruce Little field.
4. Stormy Attaway, MATLAB, A Practical Approach.



**PROJECT**

Course No: **MM 17410GE**

Total Credits: **02**

Max.Marks: **25**

End Term Exam: (2 Credits)

Max.Marks: **25**

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## APPLIED GROUP THEORY

Course No: **MM 17007GE**

Total Credits: **02**

End Term Exam: (2 Credits)

Max.Marks: **25**

Max.Marks: **25**

### **CREDIT I**

Basic concepts of relations and functions, binary operation, types and properties of functions, groups, sub-groups, normal subgroups, Cyclic groups and their properties; Homomorphism and Isomorphism, permutation groups, cosets and Lagrange and Cayley's theorems (statements only), Quotient groups and the homomorphism theorem, action of groups on sets, applications of groups through geometric patterns.

### **CREDIT II**

Alternating groups and their properties, symmetry groups in Euclidean space, motivation, isometries of n-space, the finite subgroups, representation theory, linear representations of groups, decomposing displacements, some compact lie groups and their representations, some examples of Lie groups, representation theory of compact Lie groups.

### **Recommended Books:**

1. G. Birkhoff and T. C. Bartee, Modern Applied Algebra, Mc-Graw Hill.
2. Arjeh Cohen, Rosane Ushirobira and Jan Draisma, Group theory for Maths, Physics and Chemistry Students.
3. J. A. Gallian, Contemporary Modern Algebra.
4. Surjeet Singh and Qazi Zameer-ud-Din, Modern Algebra.
5. P. M. Cohn, Lie Groups.



## BASIC GRAPH THEORY

Course No: <b>MM 17008GE</b>	Total Credits:	<b>02</b>
	Max.Marks:	25
<u>End Term Exam: (2 Credits)</u>	<u>Max.Marks:</u>	<u>25</u>

### **CREDIT-I**

Introduction of graphs, paths and cycles, operations on graphs, bipartite graphs and Konigs theorem, Euler graphs and Euler's theorem, Konigsberg bridge problem, Hamiltonian graphs and Dirac's theorem, EDT, degree sequences and their characterizations, degree sets.

### **CREDIT-II**

Trees and their properties, centres in trees, binary and spanning trees, degrees in trees, Cayley's theorem, fundamental cycles, generation of trees, Helly property, signed graphs, balanced signed graphs, Vertex connectivity, edge connectivity, Whitney's theorem, Planar graphs, Kuratowski's two graphs, Euler's formula, Incidence matrix  $A(G)$ , cycle matrix  $B(G)$ , fundamental cycle matrix  $B_f$ , cut-set matrix  $C(G)$ , adjacency matrix, matrix tree theorem, types of digraphs.

### **Books Recommended:**

1. R. Balakrishnan, K. Ranganathan, A Text Book of Graph Theory, Springer-Verlag, New York
2. B. Bollobas, Extremal Graph Theory, Academic Press,
3. F. Harary, Graph Theory, Addison-Wesley
4. Narsingh Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall
5. S. Pirzada, An Introduction to Graph Theory, Universities Press, OrientBlackswan, 2012
6. W. T. Tutte, Graph Theory, Cambridge University Press
7. D. B. West, Introduction to Graph Theory, Prentice Hall



## LATTICES AND BOOLEAN ALGEBRA

Course No: <b>MM 170040E</b>	Total Credits:	<b>02</b>
<u>End Term Exam: (2 Credits)</u>	Max.Marks:	25
	Max.Marks:	<u>25</u>

### **CREDIT-I**

Lattices: Set operations, product sets, equivalence relations, relation and ordering, partially ordered sets, chain or completely ordered sets, lattices properties, lattices and algebraic systems, sub-lattices, direct product and homomorphism, modular lattices, complete lattices, distributive lattices, complemented lattices.

### **CREDIT-I**

Boolean Algebra: Introduction, binary operations, algebraic structure, Boolean algebra, general properties of Boolean algebra, Boolean expressions, principle of Duality, Boolean algebra as a lattice, sub-Boolean algebra, direct product and homomorphism, representation theorem.

### **Recommended Books:**

1. Discrete Mathematics, Schaum's Outlines, Ind. Edition Tata McGraw-Hill Publishing Company Ltd. New Delhi.
2. A Text Book of Discrete Mathematics, Harish Mittal, Vinay K.Goyal, Deepak K. Goyal, I. K. Int. Publishing House Pvt. Ltd (2010).
3. Discrete Mathematical Structures, Kolman, Busby, Pross, Sixth Edition, PHI Laming Pvt. Ltd. (2010).
4. Discrete Mathematics, Richard Johnsonbaugh, sixth edition, Pearson Prentice Hall (2007).