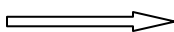


**M.A/M.Sc Mathematics Semester 2<sup>nd</sup>**

**Effective from academic session 2010**



**Repetition for 2012 with minor change**

**FUNCTIONAL ANALYSIS**

**Course No. MM-CP-205**

**BANACH SPACE:**

**Unit I**

Banach Spaces: Definition and examples, subspaces, quotient spaces, Continuous Linear Operators and their Characterization, Completeness of the space  $L(X, Y)$  of bounded linear operators (and its converse), incompleteness of  $C[a, b]$ , under the integral norm, Finite dimensional Banach spaces, Equivalence of norms on finite dimensional space and its consequences, Dual of a normed linear space, Hahn Banach theorem (extension form) and its applications, Complemented subspaces, Duals of  $l_p^n$ ,  $C_0$ ,  $l_p$  ( $p \geq 1$ ),  $C[a, b]$ .

**Unit II**

Uniform boundedness Principle and weak boundedness, Dimension of an  $\infty$ -dimensional Banach space, Conjugate of a continuous linear operator and its properties, Banach-Steinhaus theorem, open Mapping and closed graph theorems, counterexamples to Banach-Steinhaus, open mapping theorem and closed graph theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces ( $C_0$ ,  $C[0,1]$ ,  $l_p$ ,  $p \geq 1$ ), Reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, Examples of reflexive and non-reflexive Banach spaces. and examples, **HILBERT SPACE:**

**Unit III**

Hilbert spaces: Definition and examples, Cauchy's Schwartz inequality, Parallelogram law, orthonormal (o.n) systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer theorem, Gram Schmidt process, o.n basis in separable Hilbert spaces.

Fourier Series: Fourier series with respect to an o.n. base in Hilbert space, Applications to classical Fourier analysis, Examples of special o.n. bases in  $L_2[-\pi, \pi]$ . Convergence of Fourier series: Fejer's theorem on (C,1) convergence of Fourier series of a continuous function on  $(-\pi, \pi)$ , Existence of a continuous function with a divergent Fourier series at a point.

#### UNIT-IV

Projection theorem, Riesz Representation theorem. Counterexample to the Projection theorem and Riesz Representation theorem for incomplete spaces. Hilbert property of the dual of a Hilbert space and counter examples of incomplete inner product spaces, Reflexivity of Hilbert space, Adjoint of a Hilbert space operator, weak convergence and Bolzano-Weirstrass property in Hilbert Spaces. Normal and Unitary operators, Finite dimensional spectral theorem for normal operators.

#### **Recommended Books:**

1. B.V.Limaya: Funtional Analysis.
2. C.Goffman G. Pedrick: A First Course in Functional Analysis.
3. L.A. Lusternick & V.J. Sobolov. : Elements of Functional Analysis
4. J.B. Conway : A Course in Functional Analysis.