M.A/M.Sc Mathematics Semester 2nd

Effective from academic session 2010 ⟹ **Repetition for 2012 with minor change**

FUNCTIONAL ANALYSIS

Course No. MM-CP-205

BANACH SPACE:

Unit I

Banach Spaces: Definition and examples, subspaces, quotient spaces, Continuous Linear Operators and their Characterization, Completeness of the space L(X,Y) of bounded linear operators (and its converse), incompleteness of C[a, b], under the integral norm, Finite dimensional Banach spaces, Equivalence of norms on finite dimensional space and its consequences, Dual of a normed linear space, Hahn Banach theorem (extension form) and its applications, Complemented subspaces, Duals of l_p^n, C_o , l_p ($p \geq 1$), C[a, b].

Unit II

Uniform boundedness Principle and weak boundedness, Dimension of an ∞-dimensional Banach space, Conjugate of a continuous linear operator and its properties, Banach-Steinhauss theorem, open Mapping and closed graph theorems, counterexamples to Banach-Steinhauss, open mapping theorem and closed graph theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces (C_o, C[0,1], l_p, $p \geq 1$), Reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, Examples of reflexive and non-reflexive Banach spaces.and examples, **HILBERT SPACE**:

Unit III

Hilbert spaces: Definition and examples, Cauchy's Schwartz inequality, Parallogram law, orthonormal (o.n) systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer theorem, Gram Schmidt process, o.n basis in separable Hilbert spaces.

Fourier Series: Fourier series with respect to an o.n. base in Hilbert space, Applications to classical Fourier analysis, Examples of special o.n. bases in $L_2[-\pi, \pi]$. Convergence of Fourier series: Fejer's theorem on (C,1) convergence of Fourier series of a continuous function on ($-\pi$,π), Existence of a continuous function with a divergent Fourier series at a point.

UNIT-IV

Projection theorem, Riesz Representation theorem. Counterexample to the Projection theorem and Riesz Representation theorem for incomplete spaces. Hilbert property of the dual of a Hilbert space and counter examples of incomplete inner product spaces, Reflexivity of Hilbert space, Adjoint of a Hilbert space operator, weak convergence and Bolzano-Weirstrass property in Hilbert Spaces. Normal and Unitary operators, Finite dimensional spectral theorem for normal operators**.**

Recommended Books:

1. B.V.Limaya: Funtional Analysis.
2. C.Goffman G. Pedrick: A First Course in Functional Analysis.
3. L.A. Lusternick & V.J. Sobolov. : Elements of Functional Analysis
4. J.B. Conway : A Course in Functional Analysis.