Semester - IV

Theory of Numbers—II

Course No. MM-CP-403 *Maximum Marks: 100*
Duration of Examination: 3 hrs *(a) External Exam: 80*
(b) Internal Exam: 20

Unit I

$$\sum_{d|n}\phi(d)=n, \qquad \phi(m)=m\prod_{p|m}\left(1-\frac{1}{p}\right)$$

Integers belonging to a given exponent mod p and related results. If any integer belongs to t (mod p), then exactly $\phi(t)$ incongruent numbers belong to t(mod p). Primitive roots. There are ϕ (p-1) primitive roots of a odd prime p. Any power of an odd prime has a primitive root. The function λ (m) and its properties. $a^{\lambda(m)}\equiv 1(\text{mod } m)$, where (a , m)=1.There is always an integer which belongs to $\lambda(m)$ (mod m). The numbers having primitive roots are 1,2, 4, p^{α} and $2p^{\alpha}$.where p is an odd prime.

Unit II

Quadratic residues. Euler criterion. The Legendre symbol and its properties. Lemma of Gauss. If p is an odd prime and (a, 2p) =1,

$$\text{then } \left(\frac{a}{p}\right)=(-1)^{t} \quad \text{where } t=\sum_{j=1}^{(p-1)/2}\left[\frac{ja}{p}\right], \text{ and} \left(\frac{2}{p}\right)=(-1)^{(p^2-1)/8}$$

The law of a Quadratic Reciprocity, Characterization of primes of which 2,-2, 3,-3, 5, 6 and 10 are quadratic residues or non residues. Jacobi symbols and its properties.

Unit III

Number theoretic functions. Some simple properties of $\tau(n)$,$\sigma(n)$, $\phi(n)$ and $\mu(n)$. Mobius inversion formula. Perfect numbers. Necessary and sufficient condition for an even number to be perfect. The function [x] and its properties. The symbols "O", "o", and " ~" .Euler's constant γ . The series

$\sum_{p\leq n} 1/p$ diverges. Π $p<4^n$, for $n\geq 2$. Average order of magnitudes of p

$\tau(n)$, $\sigma(n)$, $\phi(n)$. Farey fractions. Rational approximation.

Unit IV

Simple continued fractions. Application of the theory of infinite continued fractions to the approximation of irrationals by rationals. Hurwitz theorem. $\sqrt{5}$ is the best possible constant in the Hurwitz theorem. Relation between Riemann Zeta function and the set of primes. Characters. The L-Function $L(S, \chi)$ and its properties. Dirichlet's theorem on infinity of primes in an arithmetic progression (its scope as in Leveque's topics in Number Theory, Vol. II. Chapter VI).

References

1. Topics in number theory by W. J. Leveque, Vol. I and II Addition Wesley Publishing Company, INC.
2. An introduction of the Theory of numbers by I. Niven and H.S Zucherman.
3. Number Theory by Boevich and Shafeviech, I.R Academic Press.
4. Analytic Number Theory by T.M Apostal, Springer international.
5. An introduction to the theory of Numbers by G.H Hardy and Wright.
6. A course in Arithmetic, by J.P. Serre, GTM Vol. Springer Verlag 1973.
7. An elementary Number theory of E. Landau.